## Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one Journal page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.

## **Comment on "Conservation Errors** and Convergence Characteristics of **Iterative Space-Marching Algorithms**"

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N Ref. 1, the authors have critically examined the new iterative space-marching algorithms for numerical solution of the parabolized Navier-Stokes equations. They claim to have shown by analysis and numerical examples that employing parabolized flux difference splitting (PFDS) in the marching direction can lead to serious conservation errors even though a finite volume formulation is employed. The authors state that the parabolized flux vector splitting (PFVS) is conservative in the telescoping sense and that the quantity that is conserved depends on the form of splitting that is employed. They also state that the PFDS scheme uses different values for the left and right face, and, hence, it is not possible to develop a PFDS scheme that is conservative. They seem to conclude that the PFVS scheme with Vigneron splitting<sup>1</sup> is better than the PFDS scheme since the former is conservative.

In a Vigneron type of flux vector splitting, only the pressure gradient term is split into a downstream and an upstream component, and when the upstream components of the fluxes are neglected in subsonic and/or reverse flow regions, the mass flux term is retained in its full form. Hence, one does not have to perform any computation to conclude that the mass flux conservation is achieved to a great degree of accuracy when such a flux vector splitting is employed.

In the case of the hypersonic inlet problem, the authors have presented figures to show only the error in the conservation of mass flux. Since a good estimate of the accuracy of a solution can be obtained only when the errors in all of the conserved quantities are determined, it appears that one cannot draw any conclusion regarding the accuracy of the PFVS method with Vigneron splitting by just looking at the conservation error in only one of the fluxes, especially the mass flux, which, due to the type of splitting employed, should be conserved exactly. All the authors have succeeded in demonstrating is that they probably do not have any coding errors.

Accumulated error depends on the streamwise spacing employed and the overall grid resolution in the marching plane. When the same problem was studied using the USA-RG2D code, two to three orders of magnitude less error in the mass flux was encountered. A time-marched solution for the same problem indicated a fairly large region of separated flow, revealing that the space-marched solution with or without accurate conservation of mass is likely to be inaccurate.

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Both the PFVS and the PFDS schemes when applied to subsonic flow problems introduce errors in the solution. For attached flows, in the PFVS scheme with Vigneron splitting, the error arises from neglecting the upstream component of the pressure gradient term in the x-momentum equation, whereas in the case of the PFDS scheme of Ota et al., it stems from neglecting the u-c component of all of the fluxes. In the limiting case of Mach number - 0, the former completely neglects the pressure gradient term, whereas the latter neglects only a part of it. Such being the nature of these two approximations, is it possible to conclude that one method is better than the other just because it conserves one of the four quantities (or five in three dimensions) exactly, even though its conservation error in the other quantities may be larger? Is it possible to conclude that a scheme that conserves mass exactly and introduces larger errors in x momentum leads to a more accurate solution than a scheme that introduces smaller error in all of the conserved variables? Also, the nature of the error depends on the problem under consideration, and, hence, it is not possible to arrive at a definite conclusion about the methods based on just one or two problems.

## References

<sup>1</sup>Thompson, D. S., and Matus, R. J., "Conservation Errors and Convergence Characteristics of Iterative Space-Marching Algo-AIAA Journal, Vol. 29, No. 2, 1991, pp. 227-234.

Ota, D. K., Chakravarthy, S. R., and Darling, J. C., "An Equilibrium Air Navier-Stokes Code for Hypersonic Flow," AIAA Paper 88-0419, Jan. 1988.

## Reply by Authors to S. V. Ramakrishnan, D. K. Ota, and S. R. Chakravarthy

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THE criticism of Thompson and Matus<sup>1</sup> by Ramakrishnan et al.<sup>2</sup> is quite interesting since none of the basic findings of Ref. 1 are disputed. The criticism appears to be based on the perception that the authors concluded that parabolized flux difference splitting (FDS) was inferior to parabolized flux vector splitting (FVS). This conclusion was never explicitly stated in Ref. 1. In fact, the question of solution accuracy is a

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